

Students' Covariational Reasoning in Solving Integrals' Problems

by Rooselyna Harini_y Fuad

Submission date: 02-Apr-2018 07:29AM (UTC+0700)

Submission ID: 939512106

File name: Harini_2018_J._Phys.___Conf._Ser._947_012017.pdf (304.46K)

Word count: 2197

Character count: 12174

PAPER • OPEN ACCESS

Students' Covariational Reasoning in Solving Integrals' Problems

To cite this article: N V Harini *et al* 2018 *J. Phys.: Conf. Ser.* **947** 012017

View the [article online](#) for updates and enhancements.

Students' Covariational Reasoning in Solving Integrals' Problems

N V Harini^{1,*}, Y Fuad¹, and R Ekawati¹

¹Mathematics Department, Universitas Negeri Surabaya, Indonesia.

*Corresponding Author: vindrin.an@gmail.com / yusuffuad@unesa.ac.id

Abstract. Covariational reasoning plays an important role to indicate quantities vary in learning calculus. This study investigates students' covariational reasoning during their studies concerning two covarying quantities in integral problem. Six undergraduate students were chosen to solve problems that involved interpreting and representing how quantities change in tandem. Interviews were conducted to reveal the students' reasoning while solving covariational problems. The result emphasizes that undergraduate students were able to construct the relation of dependent variables that changes in tandem with the independent variable. However, students faced difficulty in forming images of continuously changing rates and could not accurately apply the concept of integrals. These findings suggest that learning calculus should be increased emphasis on coordinating images of two quantities changing in tandem about instantaneously rate of change and to promote conceptual knowledge in integral techniques.

1. Introduction

Several studies [1,2,3,4,5] have revealed that the thinking of calculus students that are interpreting and representing the varying rate of change for intervals of a function's domain is slow to develop, with specific problem reported in students' ability to interpret graphical function information. Studies by [1,2,5] have underlined that calculus students do not appear to view a graph of a function as means of defining a covarying relationship between two variables. Whereas this covariation view of function has also been essential for understanding concepts of calculus [1,6].

In [7] had showed that not only covariational reasoning abilities of teachers were weak and lack depth, but also their predictions about students' reasoning abilities bounded by their own thoughts related to the problem. However, its result only focused on the level of covariational reasoning and did not investigate deeply about teachers' mental action. Moreover, many researchers [8,9,1,10] had investigated about covariational reasoning, but none of them exposed covariational reasoning that related with definite integral concepts. The definite integral concept is one of the main concepts in studying calculus. Many researchers [11,12,13,14] indicated that students have consistently difficulties in understanding and implementing the concept of definite integral, with specific problem, namely computational error and misconception in evaluate the proposed integrals. Therefore this study focuses the investigation on how students' reason during their studies concerning two covarying quantities in integral problems.



Content from this work may be used under the terms of the Creative Commons Attribution 3.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.

Published under licence by IOP Publishing Ltd

2. Method

This study was conducted on the international class of the mathematics' education study programme of Universitas Negeri Surabaya (Unesa), in academic year 2016 which was chosen by purposively sampling. The international class consists of 4 male and 21 female. From the test of represented 2nd semester calculus, we obtained that 6 students (score >80), 12 students ($80 \geq \text{score} \geq 70$), and 7 students (score <70). Six students had sign as volunteer subjects of the research. Six volunteerly students who had studied definite integrals topic in calculus class were set up to answer three items of the proposed problems relating to an analysis of covarying two quantities.

The instrument consists of three items, and had validated by two senior lecturers of the mathematics' department of Unesa. The six students invited to complete three written problems that involve interpreting and representing how quantities vary in tandem, which were identically designed in context but the problems have different representations, namely algebraic representation, graphical representation, and numerical representation. Three of these students (labeling with A, B, and C) were volunteerly invited to participate in the interview sessions that concentrated on how students reason during solving covariational problems. The selection of the interview subjects was also based on diversity responses on their written test. The analysis focused on students' use of reasoning covariationally with responses to the written test, which related with five mental actions [1] of covariational reasoning (see Table 1).

Table 1. Mental action of covariational reasoning

Mental Action	Description of Mental Action	Behaviors
Mental Action 1 (MA1)	Coordinating the value of one variable with changes in the other.	Labeling the axes with verbal indications of coordinating the two variables (e.g., y changes with changes in x).
Mental Action 2 (MA2)	Coordinating the direction of change of one variable with changes in the other variable.	Constructing an increasing straight line. Verbalizing an awareness of the direction of the change in the output while considering changes in the input.
Mental Action 3 (MA3)	Coordinating the amount of change of one variable with changes in the other.	Plotting points/constructing secant lines. Verbalizing an awareness of the variable amount of change of the output while considering changes in the input.
Mental Action 4 (MA4)	Coordinating the average rate-of-change of the function with uniform increments of change in the input variable.	Constructing contiguous secant lines for the domain. Verbalizing an awareness of the rate of change of the output (with respect input) while considering uniform increments of the input.
Mental Action 5 (MA5)	Coordinating the instantaneous rate of change of the function with continuous changes in the independent variable for the entire domain of function.	Constructing a smooth curve with clear indications of concavity changes. Verbalizing an awareness of the instantaneous changes in the rate of change for the entire domain of the function (direction of concavities and inflection points are correct).

Adopted from [1]

3. Results and Discussion

The given problem (see Figure 1) prompted students to construct a graph of a dynamic situation with a continuously changing rate. Table 2 shows the kinds of responses that the six students provided on the written test. Only one student of these high performing 2nd term calculus students given an acceptable solution, while four students constructed acceptable graph except discontinuous part and one other constructed collected of line segments. During the follow up interview to describe the graph's shape, the three interview subjects provided varied responses.

Student A provided responses that indicated all five mental actions that linear with [1]. When prompted to explain the rational for his acceptable graph (see figure 2), student A stated the following statements:

I remember the relationship of volume, debit, and time. Volume is area of debit and time and I used definite integral to find data of changing volume in each hour. Then I constructed $v(t)$ graph in the Cartesian coordinate, time in horizontal axe and volume in vertical axe (MA1). Based on my graphical sketch, I consider that $v(t)$ must contain increasing function at $t=0$ untill $t=5$ and decreasing function when $t=5$ untill $t=7$ (MA2). By using the definite integral concept, I compute the amount of change in $v(t)$ each hour (MA3). Then, I find the exact value of $v(t)$ during 7 hours. Oh but it still perform a set of points. No, it must be continuous curve since the input (time) is real number (MA4). Okay, let me try to make shorter interval when computing value of $v(t)$ using definite integrals. Finally, I get that the type of graph that presents volume during 7 hours. When $t=0$ untill $t=3$ the curve is concave since it is a quadratic function that have positive a .

Student B produced an acceptable graph except discontinuous part (see figure 3). When prompted to explain why she constructed that type of graph, she stated as follows

I just draw graph like what the result of my integral process. I check all initial and terminal points in each intervals of piecewise function, and I get that.

She appeared to have difficulty in constructing curve of continuously changing rate. Further prompting revealed that she was conceptually able to construct acceptable graph, but she lack in the process of integral. She missed the constant C in integral process, therefore her computation and plotting point did not lead to acceptable graph. This condition is similarly to that of [13].

Student C constructed a set of segments (see figure 4). When prompted to explain the rationale for her graph, she stated

Since the function of $Q(t)$ is linear so the graph of $v(t)$ should be linear". It indicated that she appeared to have difficulty to construct image of continuously changing instantaneous rate".

Table 2. Data Responses of Problem

Responses types	Number of students out of 6 providing each response type
All aspects of graph were acceptable	1
Acceptable graph, except discontinuous part	4
Collected of line segments	1

Let $Q(t)$ represent the rate at which amount of water in pool changed during 7 hours periods. Assume that the initial volume was 800 m^3 and there were two kinds of pipes those connected with the pool, a pipe that filling water to the pool and other throwing water out of pool. Construct graph of $v(t)$, given that $Q = \frac{dv}{dt}$



Figure 1. Covariational Problem

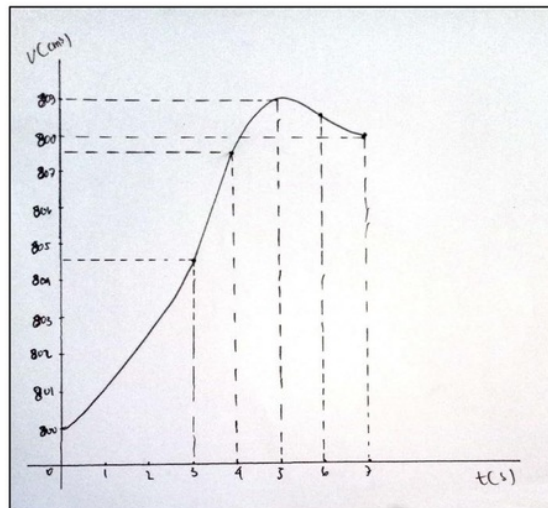


Figure 2. Graph of Student A

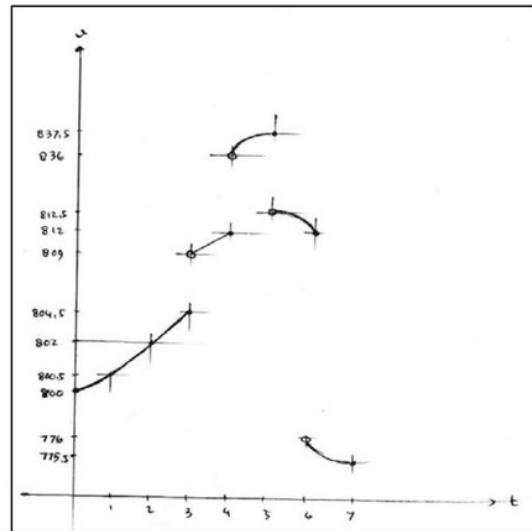


Figure 3. Graph of Student B

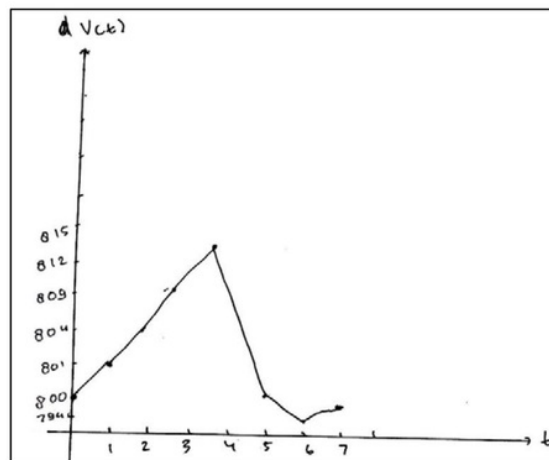


Figure 4. Graph of Student C

4. Conclusion

5

Based on the results, students were able to construct the relation of dependent variable that change in tandem with independent variable and worked on all five mental actions. Those mental actions are: coordinating the value of one variable with changes in the other, coordinating the direction of change of one variable with changes in the other variable, coordinating the amount of changes of one variable with changes in the other, coordinating the average rate of change of the function with uniform increments of change in the input

variable, and coordinating the instantaneous rate of change of the function with continuous changes in the independent variable for the entire domain of function. However, students appeared to have difficulty in forming image of continuously changing rate and could not accurately apply integral technique. This findings suggest that learning in calculus should place increased emphasis on coordinating image of two quantities changing in tandem about instantaneous rate of change and conceptual knowledge in integral technique. Further studies are needed to investigate how to develop students' reasoning in coordinating the instantaneous rate of change of the function with continuous changes in the independent variable for the entire domain of the function.

References

- [1] Carlson M, Jacobs S, Coe E, Larsen S, and Hsu E 2002 Applying Covariational Reasoning while Modeling Dynamic Events: A Framework and a Study *Journal for Research in Mathematics Education* **33** (5) 352-78
- [2] Monk S 1992 Students' Understanding of a Function Given by a Physical Model in Harel G and Dubinsky E The Concept of Function: Aspects of Epistemology and Pedagogy **25** 175-93
- [3] Thompson P W 1994 Images of Rate and Operational Understanding of the Fundamental Theorem of Calculus *Educational Studies in Mathematics* **26** (2-3) 229-74
- [4] Subanji 2011 *Teori Berpikir Pseudo Penalaran Kovariasional* (Malang: UMPRESS)
- [5] Tall D 1993 Students' Difficulties in Calculus *Proceedings of Working Group 3 on Students' Difficulties in Calculus, ICME-7 1992, Québec, Canada* 13-28
- [6] Cotrill J, Dubinsky E, Nichols D, Schwingendorf K, Thomas K, and Vidakovic D 1996 Understanding the Limit Concept: Beginning with a Coordinated Process Schema 1-23
- [7] SenZeytun A, Cetinkaya B, and Elbas A K 2010 Mathematics Teachers' Covariational Reasoning Levels and Predictions about Students' Covariational Reasoning Abilities *Educational Science: Theory and Practice* **10**(3) 1601-12
- [8] Moore K C, Paoletti T, Musgrave S 2013 Covariational Reasoning and Invariance among Coordinate Systems *Journal of Mathematical Behavior* **32** 461-73
- [9] Strom A D 2006 The Role of Covariational Reasoning in Learning and Understanding Exponential Function *Proceeding PME-NA* **2** 624-30
- [10] Zieffler A S and Garfield J B 2009 Modelling the Growth of Students' Covariational Reasoning during an Introductory Statistics Course *Statistics Education Research Journal* **8**(1) 7-31
- [11] Grundmeier T A, Hansen J, and Sousa E 2006 An Exploration of Definition and Procedural Fluency in Integral Calculus **16**(2) 178-91
- [12] Mahir N 2009 Conceptual and Procedural Performance of Undergraduate Students Integration **40**(2) 201-11
- [13] Serhan D 2015 Students' Understanding of the Definite Integral Concept *International Journal of Research in Education and Science (IJRES)* **1**(1) 84-8
- [14] Tall D O and Vinner S 1981 Concept Image and Concept Definition in Mathematics, with Particular Reference to Limits and Continuity **12** 151-6

Students' Covariational Reasoning in Solving Integrals' Problems

ORIGINALITY REPORT

29%

SIMILARITY INDEX

26%

INTERNET SOURCES

25%

PUBLICATIONS

15%

STUDENT PAPERS

PRIMARY SOURCES

1

addi.ehu.es

Internet Source

5%

2

wp.towson.edu

Internet Source

4%

3

files.eric.ed.gov

Internet Source

3%

4

pat-thompson.net

Internet Source

3%

5

asu.pure.elsevier.com

Internet Source

3%

6

A N Safitri, D Juniati, Masriyah. "Students' Relational Understanding in Quadrilateral Problem Solving Based on Adversity Quotient", Journal of Physics: Conference Series, 2018

Publication

2%

7

www.pat-thompson.net

Internet Source

2%

8

Internet Source

2%

9

N Azlina, S M Amin, A Lukito. "Creativity of Field-dependent and Field-independent Students in Posing Mathematical Problems", Journal of Physics: Conference Series, 2018

Publication

2%

10

www.maa.org

Internet Source

1%

11

www.er.uqam.ca

Internet Source

1%

12

www.dergipark.ulakbim.gov.tr

Internet Source

1%

Exclude quotes On

Exclude matches < 1%

Exclude bibliography On